Y. Ichkitidze

CYCLES OF STOCK MARKET: THEORETICAL MODEL AND EMPIRICAL EVIDENCES

Abstract *According to the efficient-market hypothesis, cycles in stock prices are only random fluctuations which are not related to economic variables. The empirical data do not confirm this assumption. A study represented in this paper demonstrates that S&P500 stock index at constant prices for the period of 1871-2013 include four approximately 30-year cycles which have statistically significant correlation with the GDP growth rate and inflation rate. Modeling of price formation in the stock market allows to make sure that the reason of cyclicality is duration of price adjustment due to long-term innovation changes in economy.*

Keywords: *stock market cycles, stock market pricing, efficient market hypothesis, business cycles, stock market forecasting, fractal market hypothesis*

1. Do Stock Prices Have Cycles?

The phenomenon of cyclicality of economic development is well known. The NBER statistical data include information on 34 business cycles observed in the period from 1854 till present[1]. Long Kondratieff waves (40-60 years), medium-term Kuznets cycles (20-30 years), shorter Juglar cycles (7-12 years, the closest to NBER business cycles), and short-term Kitchin cycles (3-5 years) are widely recognized as well. In the meantime, considering whether cyclicality is present in dynamics of stock prices (to be more exact, in stock indices), we will face a negative answer provided by the dominant scientific theory.

The efficient-market hypothesis (EMH) developed in 1960s by Nobel laureate in Economics 2013 Eugene Fama denies existence of cyclicality in stock prices. According to this hypothesis, if a cycle existed, then actions of rational market participants would remove the correlation caused by such cycle, i.e. make increments of market prices independent (Fama 1965).[2] Therefore, all cycles observed in the economic reality cannot be observed in the efficient stock market. In case some cycles are observed, these would be cycles of random nature[3] which do not correlate with other economic indicators. Hence, there is no sense in discussion of cyclicality in stock markets.

---

[1] Refer to http://www.nber.org/cycles/cyclesmain.html

[2] To be more exact, according to the efficient-market hypothesis, any correlation in the information process will be removed by rational market participants, thus making increments of market prices independent. This, obviously, implies absence of cycles (or alternating systematic tendencies) in stock prices even upon information diffusion as a result of economic development,

[3] as, according to the efficient-market hypothesis, stock prices change similarly to simple random walk.

In the meantime, empirical data point to approximately 30-year cycles in S&P500 dynamics (see Figure 1). What is this phenomenon? Does it have random nature in accordance with the efficient-market hypothesis, or is it caused by economic regularities?

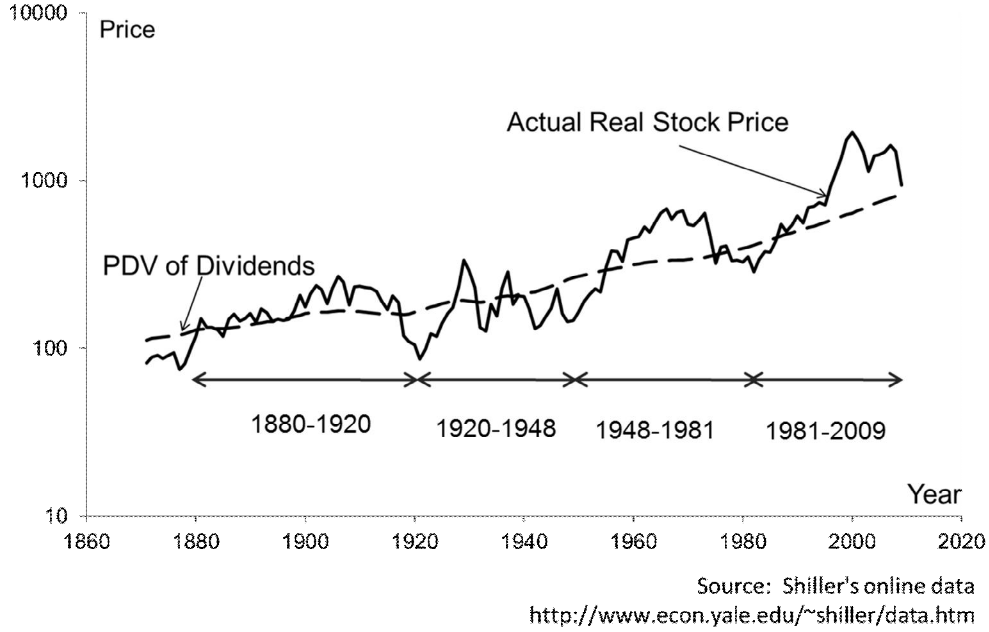

Figure 1. First Glance on Stock Market Cycles

Certain information may be obtained from evidences of famous and successful investors. Many of those do not correspond to the efficient-market hypothesis. For example, the well-known model of “boom/bust sequence”, developed by G. Soros on the basis of his empirical observations (Soros 1987), obviously, characterizes cyclic development and, therefore, contradicts the efficient-market hypothesis. Axioms of technical analysis of stock prices (which is unreasonable from the point of view of efficient markets but widely used in practice) clearly point to existence of wave dynamics in prices in correspondence with the "trend-correction" sequence. Moreover, one of its tools – Elliott wave analysis – directly characterizes existence of cyclic regularities in stock prices. It is clear that we cannot but mention the so-called line of least resistance by J. Livermore (Lefevre 1923). In essence, it characterizes systematic shift of stock prices, which forms alternating phases of stock price increase and decrease.

In general, there are enough evidences of successful experts in favor of existence of cyclicality in stock markets. When summarizing these evidences, we can state that, one way or another, they come down to irregular cycles in the stock market, occurring as a result of evolution. The cycles consist of successive opposite trends (see Figure 2). There exist tools of determination of trend break points (which are not recognized scientifically). These tools are based, first of all, on wave analysis, moving average and lines of support / resistance (see, for example, Ichkitidze, 2014).[4] Moreover, there even exist scientific models characterizing these trends, i.e. return distributions proposed by Mandelbrot (1960,

---

[4] It is obvious that these points can be determined not post factum but online, with particular uncertainty.

1961) and the subsequent fractal market hypothesis (Peters 1989) and coherent market hypothesis (Vaga 1990)[5]. However, they are not recognized scientifically and considered to be unproved.

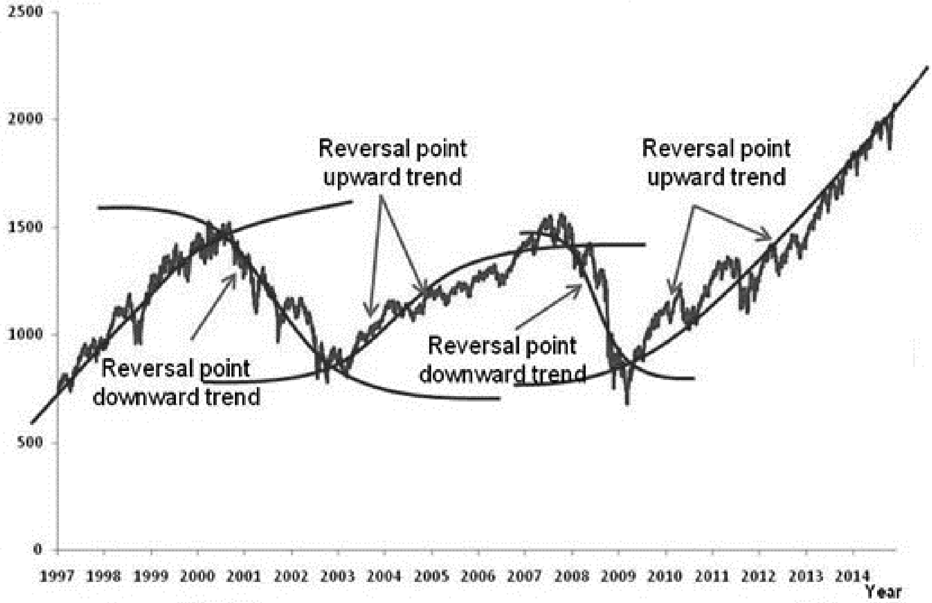

Figure 2. Irregular Cycles in S&P500 as a Result of Evolution

The basic argument in favor of insufficiency of these models is as follows. Let us assume that cyclicality (i.e. long-term oriented systematic trends which rarely change direction) in stock prices exist. Then, why a vast number of rational investors maximizing profit cannot immediately set prices in a way to remove this cyclicality? This issue has not been rationalized yet.

Thus, before analysis of empirical data, we should overcome the occurred theoretical problem and offer a rational explanation for a phenomenon of cycles (systematic trends) in the stock market.

2. Theory

2.1. Refinement of the Rational Investor's Demand Function

To offer a rational explanation for occurrence of cycles in stock prices and understand their nature (reasons of occurrence), we should thoroughly consider the price formation process in the stock market. In contrast to buyers in the consumer market, buyers in the capital market (investors) assume risks that may bring them to bankruptcy. To be more exact, as they have limited wealth and, upon attractive investment possibilities, borrow some funds, then, in case their expectations are wrong, then their creditworthiness will deteriorate and the loan rate will increase.[6] This, in its turn, will cause the response which is not observed in consumer markets (excl. the uncommon Giffen effect), i.e. price decrease will lead to decrease in demand for stocks as borrowing expenses will increase. The above can be summarized in the form of Proposition 1.

---

[5] By and large, all of them are classified as nonlinear dynamic models which are considered as feedback models and complex enough to be characterized mathematically.

[6] Or, if an investor uses the traditional scheme of marginal trading, then decrease in prices of stocks purchased will lead to decrease in his/her wealth and available credit amount.

Proposition 1. The function of demand for capital assets is different from the function of demand for consumer goods as it includes an area of direct correlation between price and demand, occurring as a result of limited wealth and borrowing conditions (see Fig. 3)

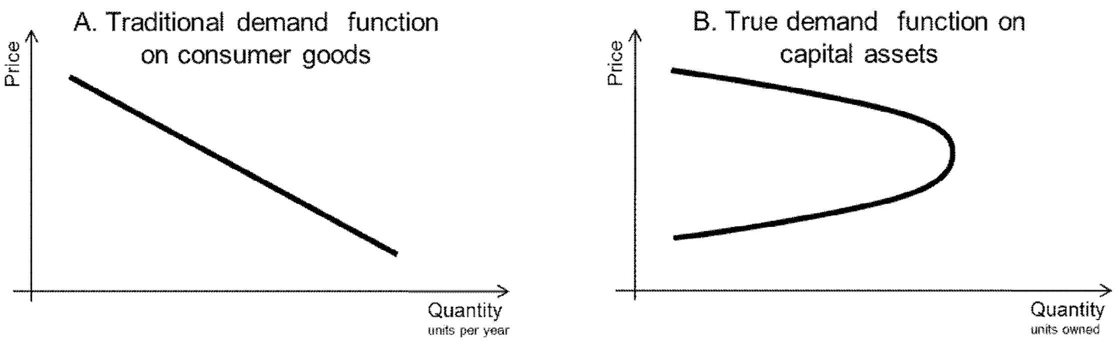

Figure 3. Functions of Demand for Consumer Goods and Capital Assets

To prove proposition 1 [7], let us consider actions of the investor in the company-specific stock market (by convention, we consider that all possibilities of portfolio diversification were used, therefore his/her "stock" is actually a stock market index or diversified portfolio).

The investor tends to:

A) correctly estimate future cash flows on stocks, and corresponding risks;

B) properly evaluate density of distribution of the stock's fair (intrinsic) value;

C) take into account his/her own risk-averse behavior and use the expected utility function in decision-making process.

The investor has the possibility to buy/sell stocks at market prices. His/her task is to determine the optimal demand for stocks under current conditions.

The investor solves this task as follows:

1. he/she maximizes the average value of the expected utility function;

$$\overline{U}(\tilde{w}) \to \max \qquad (1)$$

2. since the investor has limited wealth, then if he/she wants to purchase stocks for the sum exceeding his/her wealth, he/she should borrow[8];

3. Investor's expected wealth is formed as the sum of the initial risk-free wealth and the expected return on stocks at time zero minus the current expenses for debt servicing[9] (Equation 2).

$$\tilde{w} = w_0 + c \cdot (\frac{\tilde{P}_f}{P_m} - 1) - (c - w_0) \cdot (\frac{r_d}{r_0} - 1) \qquad (2)$$

---

[7] This proving is a result of my PhD thesis presented in 2004. It was published for the first time in a Russian journal in 2006. For details, please, refer to (Ichkitidze 2006).

[8] It is necessary to mention that this task is similar to the study of Leland (1980) except this possibility of borrowing.

[9] Equation 2 assumes that the investor holds stocks and refinances its debt indefinitely. Changing of this assumption does not affect the results significantly.

where

w_0 – initial investor's wealth;

c – investor's demand for stocks (USD);

$\widetilde{P}_f$ – distribution of the stock's fair (intrinsic) value;

P_m – stock's market price;

D – investor's debt (i.e. $D = c - w_0$)

r_d - investor's debt interest rate, incl. expected default risk; [10]

r_0 - risk-free interest rate.

It is assumed that "short sales" of stocks are prohibited (i.e. $c \geq 0$) as it slightly facilitates calculations. This assumption does not affect the conclusions drawn from the solution of the task.

Thus, the task of calculating the optimal volume of rational investor's demand for stocks (C_{opt}) is complete and can be represented in the following form:

$$\overline{U}\left(w_0 + c\cdot(\frac{\widetilde{P}_f}{P_m} - 1) - (c - w_0)\cdot(\frac{r_d}{r_0} - 1) \right) \to \max_C \qquad (3)$$

Or, it can be represented with the aid of Stieltjes integral as follows:

$$\int_0^\infty U\left(w_0 + c\cdot(\frac{\widetilde{P}_f}{P_m} - 1) - (c - w_0)\cdot(\frac{r_d}{r_0} - 1) \right) dF(\widetilde{P}_f) \to \max_C \qquad (4)$$

where

$F(\tilde{P}_f)$ - distribution function of fair value.

It is necessary to point out that if the investor has some stocks in property, then his/her wealth is a function of the market price as is represented in Equation 5.

$$w_0 = w_0^i + \frac{C_{opt0}}{P_{m0}}(P_m - P_{m0}) \qquad (5)$$

where

w_0^i – initial investor's wealth, when he/she had not stocks in the property;

P_{m0} – stock's initial price, when he/she bought stocks for the first time;

C_{opt0} – investor's optimal demand for stocks (USD), when he/she bought stocks for the first time (calculated as the solution of Equation 3 with w_0^i and P_{m0})

w_0 – initial investor's wealth, when he/she has stocks in the property (incl. the value of the stocks at market price);

P_m – current stock's market price;

Given this, the solution of Equation 4[11], i.e. the resulting dependence of the rational investor's optimal demand for the stock (c_{opt} / P_m) on the market price (P_m)

---

[10] To determine r_d, I have used the Merton (1974) approach; the exact formula is $r_d = r_0 + put(1 + r_0)/D$, where *put* is the cost of the European put option for the underlying asset V (i.e. the value of the investor's assets) with the strike equal to $D\cdot(1 + r_0)$ and the expiration within one year.

[11] To solve this equation, I used computer calculations.

for the case with a risky debt, is shown in Figure 4. In order to better understand the findings, the dependence is represented for two cases: a) when the investor initially has no stocks in property (i.e. $c_{opt0} = 0$), and b) when the investor has some (X) stocks in property.

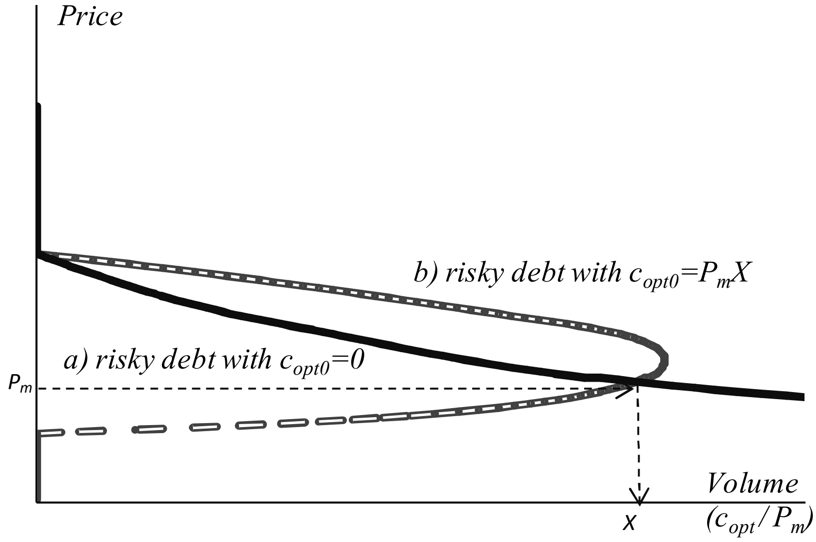

Figure 4. Classical and reflexive areas of the rational investor's demand function

Figure 4 affects our idea of rational investor's function of demand for stocks. In addition to the classical area of the demand function with inverse dependence between stock's price and investor's demand, a second area with direct dependence occurs as a result of limited wealth and borrowing possibility.[12] The first area occurs due to the fact that changes in price lead to changes in expected profit, and it corresponds to the traditional opinion. And the second area occurs due to the fact that the price begins to affect investor's creditworthiness significantly, allowing him/her to increase demand if the price moves as he/she expects, or, on the contrary, forcing him/her to reduce demand if the price goes in the opposite direction[13]. I propose to name the second area of the demand function as the "reflexive area", because the first who discovered its existence was G. Soros. Here is how he described it: "the loan may affect the value of the underlying collateral: it gives rise to a reflexive process" (Soros 1987). Of course, he means that the loan creates demand for capital assets, and that affects prices, then the prices effect creditworthiness. This, in turn, increases the amount of the loan and, therefore, increases the demand again. Describing this recursive relationship, Soros used the term of "reflexivity".

[12] A paper of Yuan (2005) also provides evidence of existence of the second area of the demand function, in which it was obtained as a result of empirically existing borrowing constraints.

[13] To be more exact, in the studied demand function, the market price affects demand via two above methods simultaneously. From the one part, increase/decrease of the market price leads to decrease/increase of the expected profit (P_f/P_m) and, therefore, decreases/increases the optimal volume of demand.
On the other part, increase/ decrease of the market price causes increase/ decrease of the investor's wealth and decreases/ increases the debt interest rate which, in its turn, increases/ decreases investor's optimal demand for the stock. The classical area of the demand function appears when the first impact is stronger than the second one. And, vice versa, when the second impact is stronger, the investor operates in the reflexive area of the demand function.

As arguments given by Fama (1965) in favor of "instantaneous adjustment" of prices do not take into account limitations of wealth of professional arbitrageurs, or, in other words, do not take into account influence of the reflexive part of the demand function on market price formation, they can be insufficient if such influence really exists.

2.2. "Big Shift" Development

Further development of the theory for explanation of stock market cycles is related to diffusion of information due to the innovation process. It is known that basic changes in the world around are caused not by separate entries of exogenous information, but by long-term processes of innovation changes characterizing macrosystem evolution (see Hirooka, 2006). Their distinctive feature is so-called "resonance effect", i.e. mutual interaction of these processes which makes their consequences not only extensive, but also difficult to forecast. Followers of the efficient-market hypothesis, as a rule, ignored influence of these processes on price formation, considering, to a greater extent, influence of some piece of information taken out from the historical context. It is not surprising that they got confirmation of their hypothesis, as prices had already taken into account this information prior to the moment of its appearance.

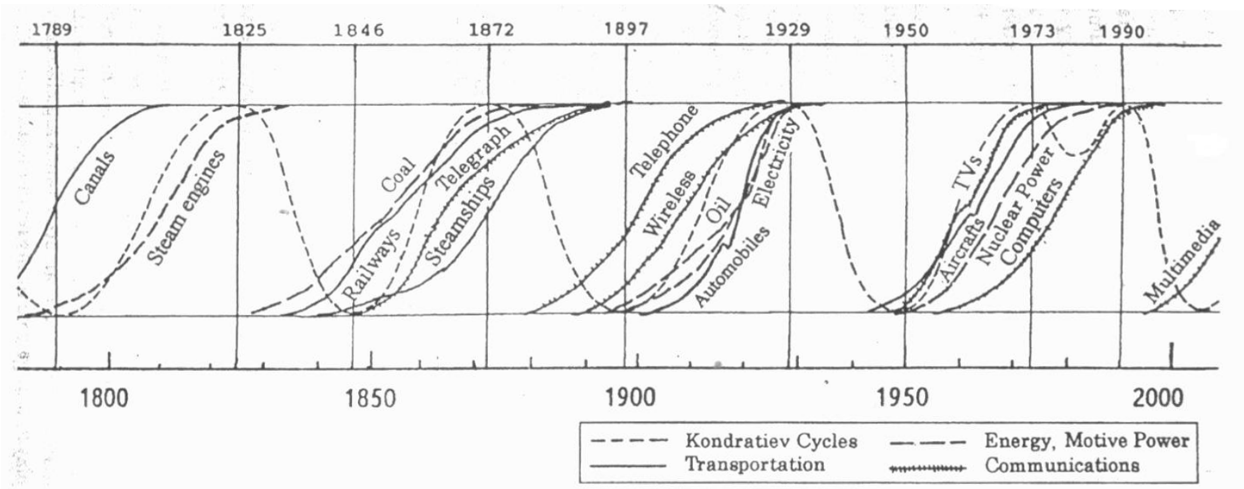

Figure 5. Hirooka's S-type Trends in Trunk Innovations

Figure 5 includes the used data on innovation S-type trends causing evolutionary changes. It is clear that a significant cluster of trunk innovations falls in 1950-1970s, computer development falls in 1980-1990s, and multimedia technologies have been developing since the beginning of 2000. These trunk innovations laid the foundation of infrastructure changes and gave momentum to diffusion of information, which, in its turn, affected expectations of investors with regard to future dividends. Thus, if we, considering this, assume that expectations are based on the existing information, then the following proposition shall be valid.

Proposition 2. While a company generates innovations, the present value of expected stock's dividends, based on available information, changes over time as an S-type function (see Figure 6) [14]

It is obvious that the present value of future dividends, defined in such manner, is subject to shift over time and, therefore, it cannot be considered as rationally expected. To get such rational present value, an investor should take into account not only the existing information, but also correlation (tendency) typical for its diffusion. Thus, expectations, generated both on basis of the existing information and its development, will change significantly as soon as an investor acknowledges the latter. "Big shift" (Figure 6), i.e. significant changes in rational expectations over a short time under influence of innovation process, will occur. Empirically, it is similar to the sunlight in the darkness as new consciousness breaks through to participants.

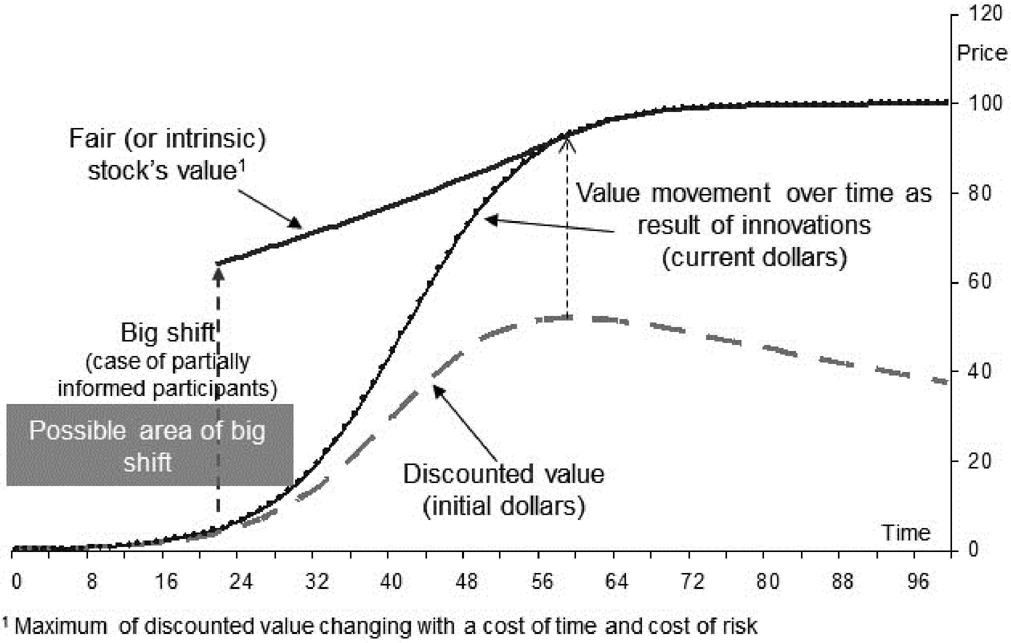

Figure 6. Occurrence of the "Big Shift" in Fair Value

As the efficient-market hypothesis states that any correlation in information entry will be taken into account by market participants in actual prices (Fama 1965), we should check whether this statement works in case of long-term (several years) information diffusion (under influence of innovation process), which results in significant changes in fair value.

2.3. Model of Stock Price Behavior over Long Distance

To check this statement, let us study price formation in the stock market under conditions of the "big shift". A graphic model of changes in market equilibrium with account for a reflexive area of the demand function is represented

[14] With regard to the stock market in general, proposition 2 shall take the following form: *trunk innovations generate the same S-type trend in stock price index due to the resonance phenomenon between separate companies*

in Figure 7. Those investors, which one of the first recognize changes (A type), act in the reflexive area of the demand function, and the rest, which do not recognize changes (B type), act in the classical area of the demand function (see Figure 7). While more and more traders recognize changes, an impulse changing market equilibrium occurs. When the number of traders, which recognized changes, exceeds the critical level, equilibrium shifts to P_{fair_A} (Figure 7). Similar changes may occur if fair value decreases significantly.

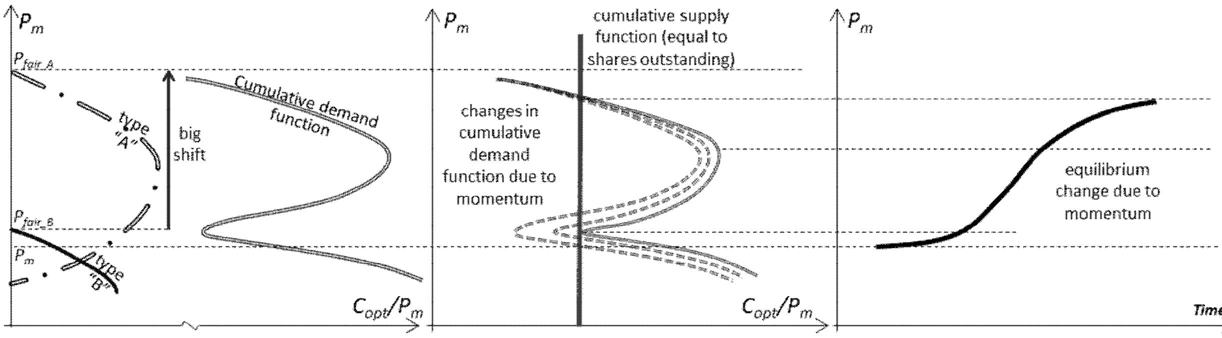

Figure 7. Model of Stock Price Behavior over Long Distance

The model formula is represented below. Equilibrium condition is zero growth of aggregate demand (Equation 6), resulting in the following: upon the specified price (P_0), known number of traders of A (α) and B types (β), and known functions of individual demand ($f_\alpha(P)$ for A type and $f_\beta(P)$ for B type)[15], market price changes (ΔP) near price P_0 are determined by impulse (λ), i.e. number of traders transferred from type B to type A (Equation 7).

$$\alpha \cdot f_\alpha(P_0) + \beta \cdot f_b(P_0) - (\alpha + \lambda) \cdot f_\alpha(P_0 + \Delta P) - (\beta - \lambda) \cdot f_b(P_0 + \Delta) = 0 \qquad (6)$$

$$\Delta P = \varphi(\lambda) \qquad (7)$$

It is obvious that the reason for absence of "instantaneous adjustment" of model prices is long-term process of recognition of the "big shift" by market participants upon limited funds. For comparison, in the model without a reflexive area, recognition of the "big shift" by narrow group of experts would lead to "instantaneous adjustment" of prices. Now they are limited by wealth size, and, for price adjustment, the "big shift" should be recognized by wider range of investors. Thus, we can state that in some cases shift of prices to new level of fair value may take not only several days or months, but years. In my opinion, such situation was observed in the US stock market during the economic cycle of 1983-1990 when we faced new phenomenons, i.e. crash of 1987, popular strategies of portfolio insurance and development of noise trading. The "big shift" related to major innovation and infrastructure changes in economy, which have "resonance effect" difficult to forecast, occurred approximately in 1984-1985. Those market participants who recognized it in time faced limited wealth, others (the majority) recognized extents of changes with significant delay. Approximately by the end of

[15] We assume that these functions are homogeneous, i.e. they differ only in expectations with regard to fair value.

the business cycle, i.e. in the beginning of 1990, prices came to new level of fair value1.

Thus, the model demonstrates that, under conditions of significant changes in fair value as a result of long-term information diffusion caused by innovation process, prices cannot immediately represent the existing correlation. Anticipatory price adjustment occurs but it is not sufficient to consider complete removal of the correlation.

Figure 8 represents a model result forecasting the S-type trend in stock's market prices, occurring as a result of long-term information diffusion due to innovation process. It has three areas characterizing various states of the stock market. The first area is related to imbalance due to bounded rationality, i.e. professional investors are late to recognize correlation in information entry. The second area is related to imbalance due to lack of funds with those investors which recognized this correlation. The third area characterizes conditions of an efficient market when prices fully reflect the existing information on innovation process and their trend is defined by cost of time and cost of risk.

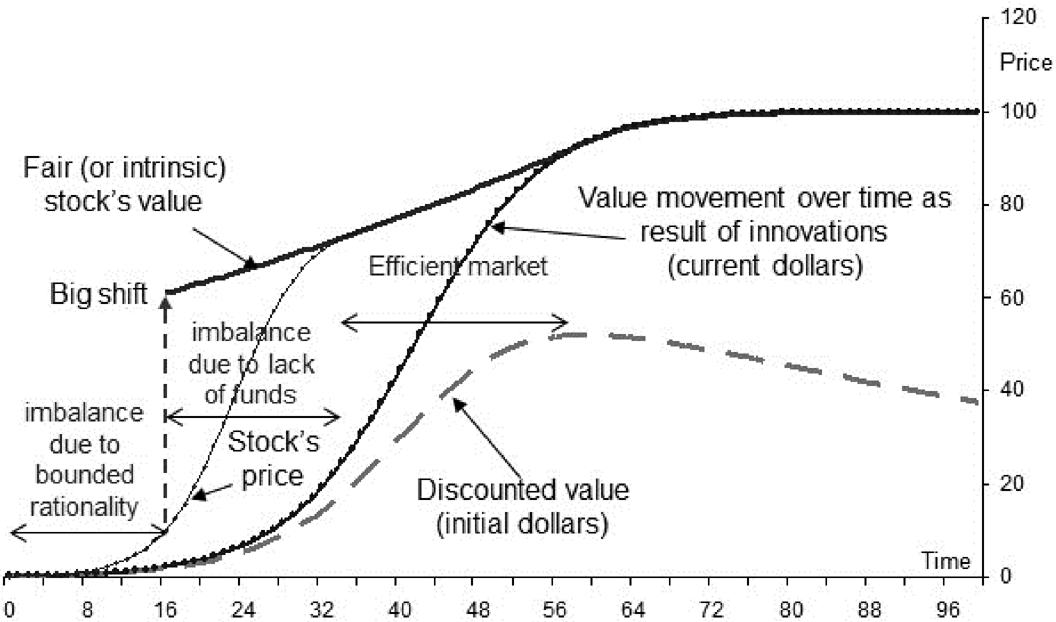

Figure 8. S-type Trend in Stock's Market Price

Now, with account for the model, we have reasons to consider cycles in the stock market not as a result of random events, but as a result of information diffusion under influence of innovation process. Having overcome objections of followers of the efficient-market hypothesis, we can begin empirical studies.

3. Evidences

3.1. Trend Following Strategies and Equity Premium Puzzle

First of all, we should note that the built theoretical model is compatible with two widely known empirical phenomena which were not rationally explained in the efficient-market hypothesis. The first of them is popularity of so-called strategies of positive feedback (strategies of trend-chasing) among investors. Their roots are traced back to the "golden rules" of David Ricardo (Grant J. 1838) and "line of least resistance" by J. Livermore (Lefevre 1923). Moreover, they can be observed in the ideology of G. Soros. Thus, the fact that we assume those as non-rational behavior (see DeLong et al., 1990) and do not consider them as a phenomenon for studies looks strange. The considered model implies that such behavior of traders is rational enough as systematic trends (cycles) in stock prices, caused by long-term process of information diffusion, occur. Thus, this phenomenon can be recognized as an empirical evidence in favor of existence of imbalance and occurrence of trend (cycle) processes in stock prices.

The second phenomenon is a so-called equity premium puzzle. According to this puzzle, actual return on stocks for the period of 1889-1978 was, in average, by 5.8% per annum more than it was forecast by the model of rational behavior (Mehra, Prescott 1985). This phenomenon is also compatible with the considered model as, upon occurrence of long-term imbalance, equity premium should be higher. Figure 9 represents the appropriate behavior of the market under conditions of the "big shift" if "instantaneous adjustment" is observed and the market is efficient, as well as the market behavior under conditions of imbalance. In both cases increments of market prices will be independent. The main difference is stock return level in the long-term; in case of an efficient market it should correspond to forecast of rational model, but in case of long-term imbalance it should exceed them. In other words, the equity premium puzzle empirically confirms that long-term imbalance in the stock market really occurs. Therefore, we can expect that cyclic processes in dynamics of stock prices will be observed.

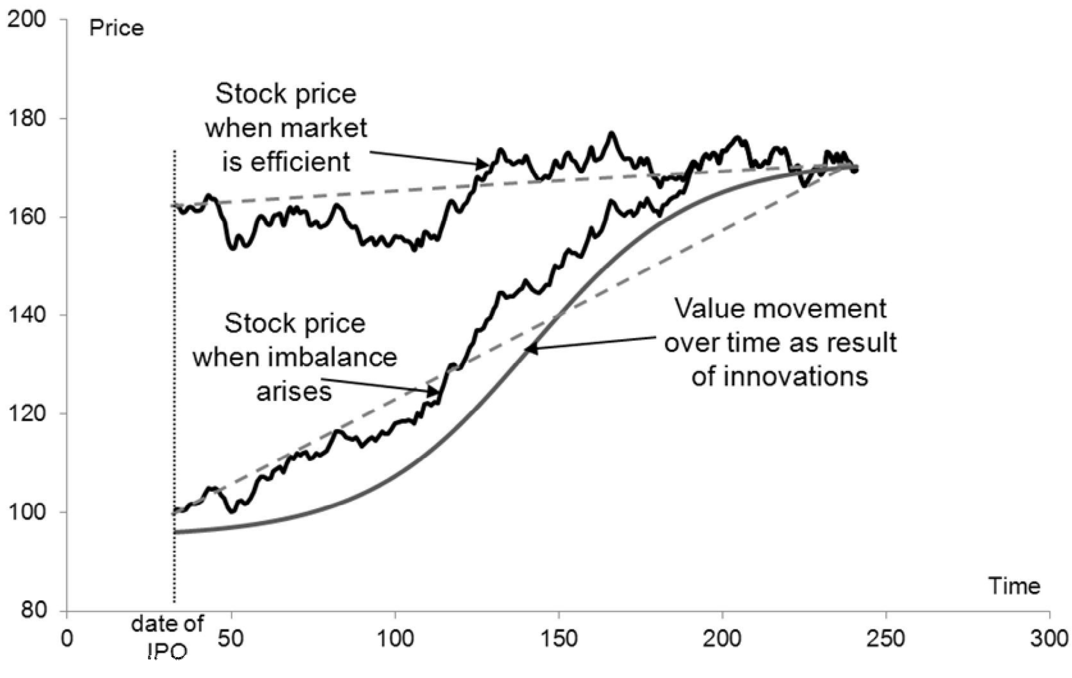

Figure 9. Differences in Stock Price Dynamics in Efficient and Imbalanced Markets

3.2. Fourier Analysis for Stock Prices

To characterize cyclicality in S&P500 series for the period of 1889-2010, let us run harmonic analysis using the simplest cyclic model of the following form $y = a \cdot \sin(T) + b \cdot \cos(T) + c$.

Let us build a series of chain growth rate with regard to real values of S&P500 for 10 years from the Shiller data base[16]. Let us then define cycles of the first order. The maximum value of the determination coefficient is observed with the cycles of the period of 32 years (R^2=0.38). Let us define the cyclical component in the obtained series of remainders under the additive model once again. Now the maximum value of the determination coefficient is observed with the cycles of the period of 40 years (R^2=0.13). Finally, one more type of cycles (65 years) can be defined in the series of third remainders (after removal of two previous cycles). Its determination coefficient equals to 0.09.

The general cyclicality model with regard to the studied indicator is represented in Figure 10. In dynamics of stock prices, we can observe four cycles with the period of approximately 30 years. The growth phase of the first cycle corresponded to growth of the stock market in 1897-1906. The second cycle (1921-1947) had the clear growth phase falling in 1924-1929. The third cycle (1947-1979) was characterized by growth in 1949-1965. Finally, the fourth cycle observed in 1979-2010 had the growth phase falling in 1984-1999. It is obvious that these 30-year cycles can be classified as Kuznets cycles in dynamics of stock prices. They do not disappear upon analysis of nominal return.

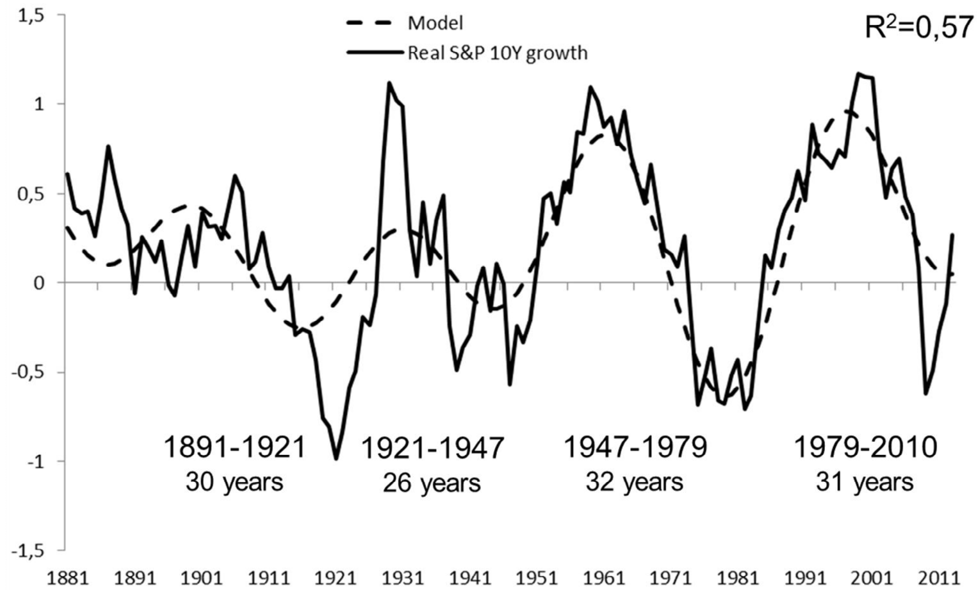

Figure 10. Kuznets Cycle in Stock Prices

To demonstrate empirically that there is more to these cycles than random oscillations, it is necessary to make sure that the defined cycles significantly depend on the observed economic variables.

3.3. Two Factor Model of S&P500

Let us study correlation between real return of S&P500 index for 10 years with 10-year growth rate of real GDP and CPI. In accordance with the prognoses of the efficient-market hypothesis, changes in stock prices, which completely represent all available information, should not have any significant correlation with the observed economic variables. An attempt to build a linear two factor model on

[16] See http://www.econ.yale.edu/~shiller/data.htm

the basis of all existing data, which would be statistically significant, appears to be ineffective. The model is mathematically characterized by Formula 8, and its result is represented in Figure 11. The model has low determination coefficient (0.12). Among its parameters, only a parameter of regression upon inflation appears to be significant (t-statistics 6.45).

$$Z_t = 0{,}115 \cdot X_t - 0{,}589 \cdot Y_t + 0{,}314 \qquad (8)$$

where:

Z_t - model of real return of S&P500 index for 10 years preceding t moment;

X_t - growth rate of real GDP for 10 years preceding t moment;

Y_t - growth rate of CPI for 10 years preceding t moment;

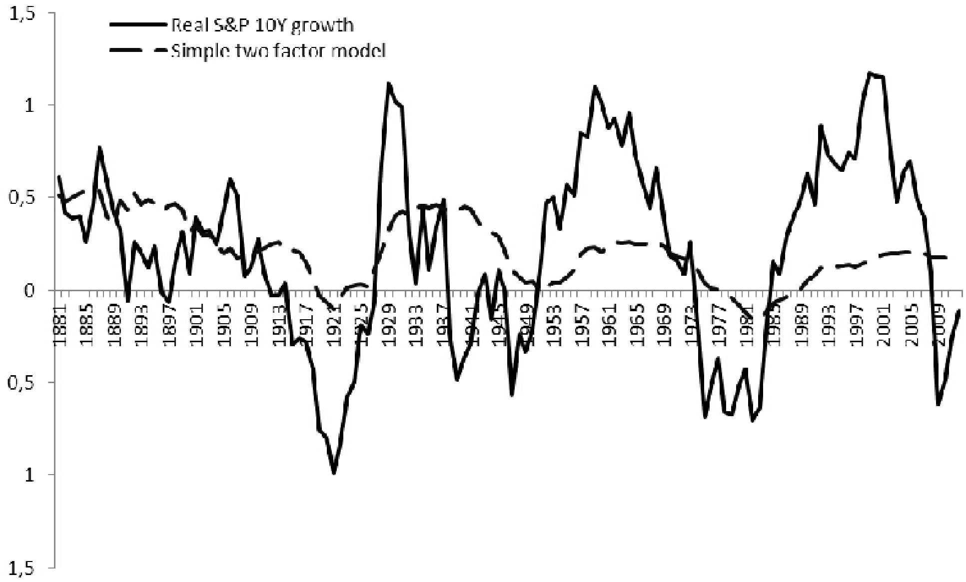

Figure 11. Simple Two Factor Model of S&P500

However, if we divide the whole data interval into two periods (1881-1940 and 1941-2010) and optimize model parameters in each period separately, then model quality will increase significantly.

The model before 1940 inclusively can be characterized by Formula 9.

$$Z_t = 1{,}347 \cdot X_t - 1{,}014 \cdot Y_t - 0{,}034 \qquad (9)$$

The determination coefficient of the model equals to 0.5, t-statistics for the GDP parameter equals to 4.46 and for the inflation parameter it equals to 7.5. Upon the table value of the Student's t-test equal to 2.66 (test significance – 0.01, 57 freedom levels), it points to significance of both determined regression coefficients.

The model after 1940 is characterized by the lag in the inflation indicator, i.e. return on stock index is 4 years ahead of changes in CPI and is represented by Formula 10.

$$Z_t = -0{,}147 \cdot X_t - 2{,}396 \cdot Y_{t+4} + 1{,}291 \qquad (10)$$

The determination coefficient equals to 0.76, t-statistics for the GDP parameter is low (0.94) and for the inflation parameter it is sufficient to be considered significant (11.61).

The combined two factor model is graphically represented in Figure 12.

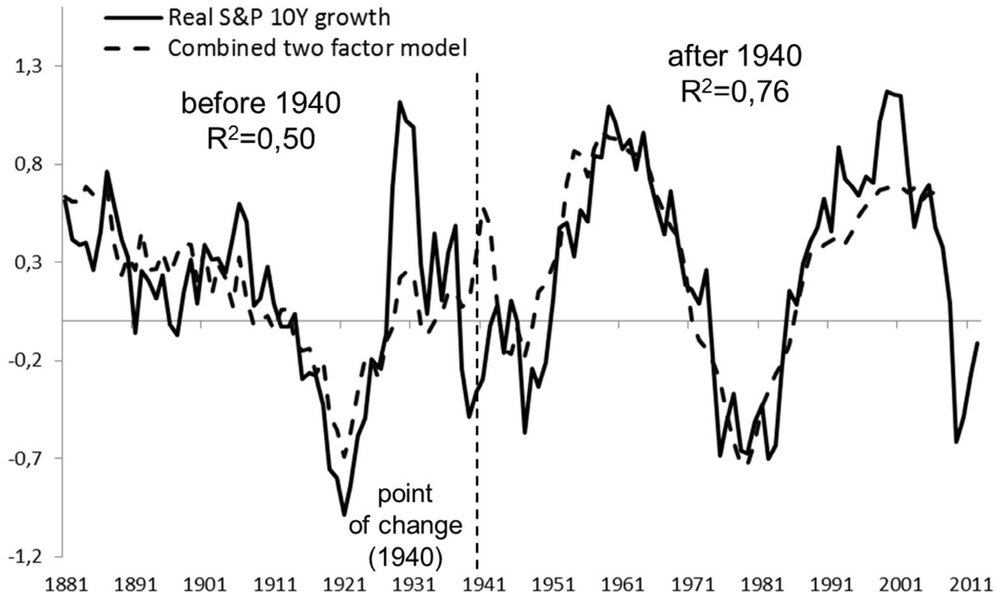

Figure 12. Combined Two Factor Model of S&P500 with the Break Point in 1940

It seems that division of data into two periods is caused by infrastructure changes occurred in the US economy around 1940, in particular, changes in economic policy, following overcoming of the Great Depression and development of monetarism. Under conditions of continuous evolution of the whole economy, changes in parameters of the model after 1940 cannot be a cause to refuse the correlation.

Thus, statistically significant correlation between cycles of the stock market and oscillations of basic economic indicators (GDP and inflation before 1940, inflation only after 1940) is proved[17], and it empirically confirms that cyclic changes in stock prices are not random. However, consistency of this fact with the efficient-market hypothesis is still unclear.

3.4. Forecasting Notes

Cyclicality of economic indicators shall not be considered as a simple deviation from the basic trend. On the contrary, it should be realized that the cycle forms the trend.[18] The analysis performed shows that dynamics of real values of S&P500 for the period 1871-2013 was formed as a result of subsequent occurrence of four, approximately 30-year, cycles which shall be considered as representation

[17] The defined correlation cannot be refused on the ground that it is real and not nominal return that is the effective attribute. The equivalent model resulting in nominal return of S&P500 index provides lower determination coefficient but demonstrates that before 1940 the growth rate of real GDP was an effective attribute, and after 1940 statistically significant correlation is observed due to growth rate of CPI.

[18] It was Schumpeter (Schumpeter, 1984) who proved this when he studied the phenomenon of economic development and described the conjuncture cycle. The mathematical model of long-term economic growth with account for cyclic oscillations is represented in various works (Akaev, 2007, 2008).

of the phenomenon of economic development in stock prices. Correct forecasting of the 5th cycle dates will allow to make investments more effectively. The game is worth the candle as at previous four growth phases the average real return of S&P500 was equal to 6.5 per annum while at the decline phases the annual average return was equal to minus 7.0%.

The model of cyclicality of the stock market, similar to that in Figure 10, can be build on the basis of natural logarithm of the real value of S&P500 (see Figure 13). It is convenient for forecasting. It confirms that the new 5th cycle began in 2014 and will last till 2045 (its growth phase will end in 2033).

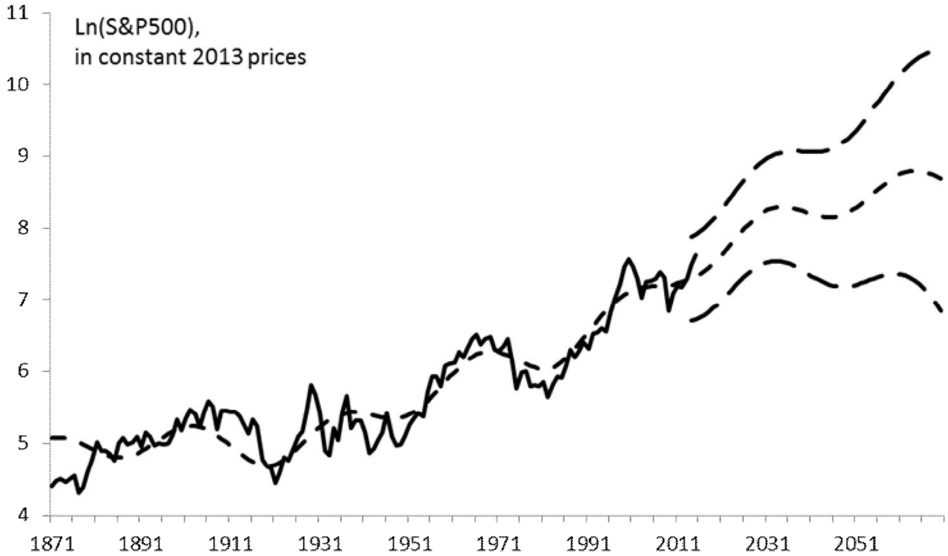

Figure 13. Natural Logarithm of S&P500 in Constant 2013 Prices and its Forecast

However, it shall be taken into account that the current 4th cycle had more extended growth phase than previous cycles (25 years instead of 18), in particular, due to infrastructure changes. Therefore, we have strong expert evidence to consider that the 4th cycle has not ended yet.

First of all, history analysis demonstrates that alteration of the defined 30-year cycles falls in the period of milestone events. The end of the first cycle fell in the World War I; the end of the second cycle fell in the Great Depression in the USA and the World War II; the end of the third cycle fell in the collapse of the Bretton Woods monetary system. What events will characterize the end of the fourth cycle? In my expert opinion, the crisis of 2008 only set off the decline phase of the fourth cycle. And countermeasure of economic policy just extend this period. Now, in 7 years after the beginning of the crisis, the chain of changes caused has been still expanding.

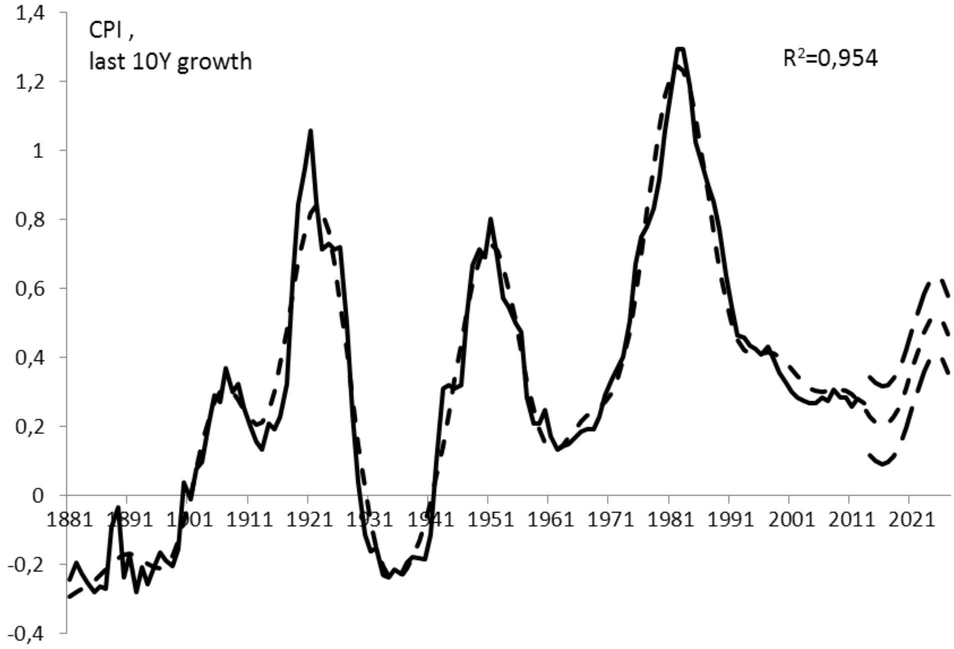

Figure 14. Inflation Cycles in the USA

Secondly, the built combined two factor model of real return of the stock index (see Formulas 9, 10 and Figure 12) points to the fact that the fourth cycle will end upon inflation acceleration. And we know that it did not happen after the crisis of 2008. Analysis of 10-year growth rate of CPI (see Figure 14), performed with the aid of cyclicality models, forecasts inflation growth in the period of 2018-2025[19]. In this period average US inflation can amount to 5% per annum. With the aid of the two factor model we can see that such inflation growth will lead to decrease of real return of S&P500 index to zero (see Figure 15) and, therefore, to the end of the fourth cycle. Then, dates of the new fifth cycle of the stock market will be related to the period after 2025.

This, with the account for expert adjustment of duration of the current decline phase of the fourth cycle to 2025, we will obtain forecast of S&P500 natural logarithm in fixed prices of 2013 as is shown in Figure 16 (confidence probability - 0.95). It is obvious that in comparison with the forecast under the model of random walk (compatible with the efficient-market hypothesis), this forecast describes future price trajectory more accurately. When it holds up, it will be considered as an additional argument in favor of forecasting technique with regard to stock indices on the basis of cyclic processes.

[19] The inflation model includes interaction of six types of cycles with various duration (15, 20, 25, 32, 41 and 68 years). Its determination coefficient is equal to 0.954.

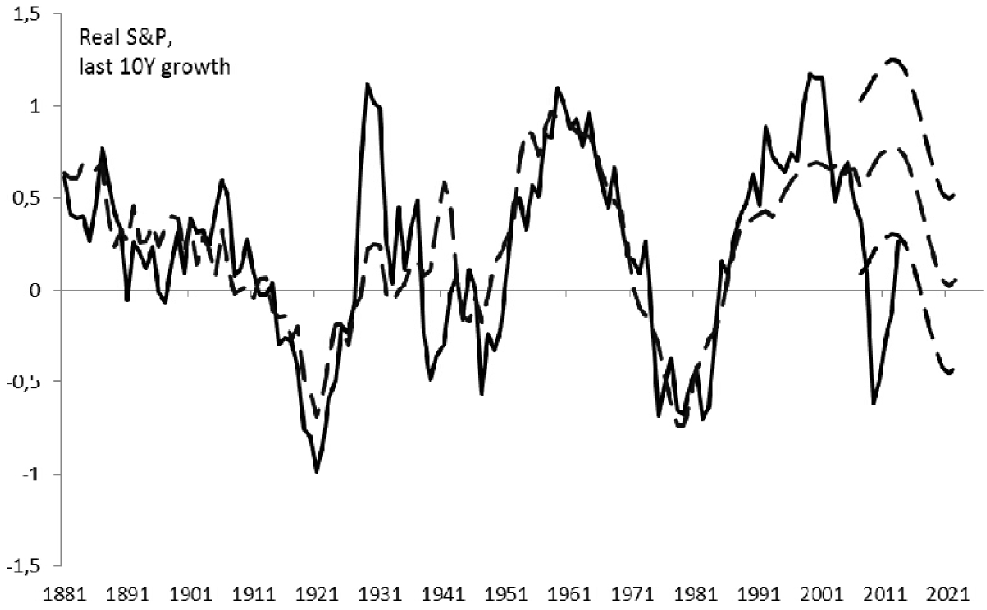

Figure 15. Forecast of Last 10Y Return of S&P500 Based on the Combined Two Factor Model

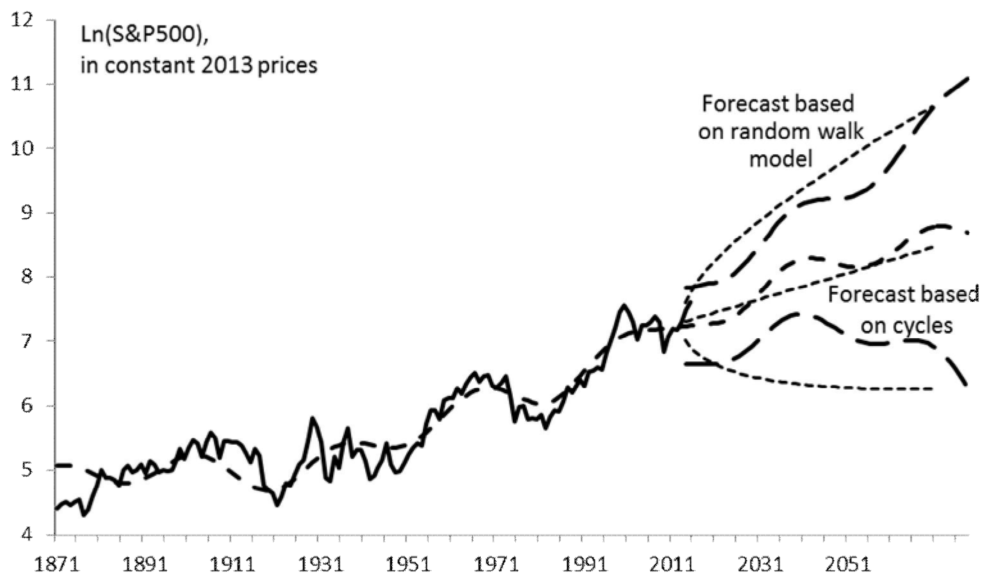

Figure 16. Final Forecast of Natural Logarithm of S&P500 in Constant 2013 Prices with the Account for Expert Adjustments

The built model implies that in the present time (or the nearest future) investors are to seek inflation guard assets. It is quite likely that the trend to growth, that has accelerated with regard to US stocks since 2011, will be overlapped by the next inflation as the build two factor model implies zero real return for the decade preceding 2021. We have time till occurrence of first indications of accelerating inflation. As soon as the occur, the market can response nonlinearly. Possibility of such scenario is additionally confirmed by high level of the monetary base, appeared in the USA after the crisis of 2008.

In the meantime, it is too early for the beginning of the new fifth cycle of the stock market, although it is showing in the distance.

Conclusion

The arguments made in this paper point to the fact that cyclic processes in the stock market exist; their registration and analysis allow to better understand the current phase of evolution of the economic system, make forecasts and substantiate the decisions made. In the course of the study four approximately 30-year cycles in S&P500 index in the period of 1871-2013 were defined. Their growth phases are related to the following periods: 1897-1906 (first cycle), 1924-1929 (second cycle), 1949-1965 (third cycle), and 1984-1999 (fourth cycle). According to the above theoretical justification, these growth phases occur as a result of significant changes in fair value due to long-term innovation process. Instantaneous price adjustment does not happen due to differences in scale: the model of price formation shows that long-term process of "big shift" recognition and limited wealth of professional arbitrageurs lead to the fact that prices repeat the process of information diffusion only with slight advance.

That fact that the defined cycles are not random and have significant correlation with economic indicators was confirmed empirically with the aid of the build two factor model. Evolution of the economic system determined the point of data division into two periods: before 1940 and after 1940. During the first period, the statistically significant correlation is observed between real return and both growth rate of real GDP and CPI. During the second period, the significant correlation is observed only with inflation. Upon analysis of nominal return of S&P500 this correlation almost does not change.

The made forecasts show that the period before 2025 will be related to the end of the fourth cycle of the stock market as during 2018-2025 inflation acceleration in the USA up to 5% per annum is possible, then the fifth wave of growth in the stock market may occur only after the end of the current phase of changes.

References

Akaev AA (2007) Vyvod obshchego uravneniya makroekonomicheskoy dinamiki, opisivaushchego sovmestnoe vzaimodeystviye dolgosrochnogo rosta i delovyh tsiklov [Derivation of the general macroeconomic dynamics equation describing the joint interaction of long-term growth and business cycles]. – Doklady Akademii Nauk 417(4):439–441 (in Russian)

Akaev AA (2008) Analiz resheniy obshchego uravneniya makroekonomicheskoy dinamiki [Analysis of solutions of the general macroeconomic dynamics equation]. Ekonomika i matematicheskiye metody [Economics and mathematical methods] 44(3):62–78 (in Russian)

De Long JB et al (1990) Positive Feedback Investment Strategies and Destabilizing Rational Speculation. J. Finance 45:379–395

Fama E (1965) The Behaviour of Stock Market Prices. J. Bus. 38:34–105

Grant J (1838) The Great Metropolis.E.L. Carey & A. Hart, Philadelphia

Hirooka M (2006) Innovation Dynamism and Economic Growth. Edward Elgar Publishing, Cheltenham

Ichkitidze YR (2006) O refleksivnosti finansovogo rynka [On reflexivity of the financial market]. Finances and business 1:25–38

Ichkitidze YR (2014) Tekhnicheskiy analiz, torgoviye taktiki, printsipy prinyatiya resheniy [Technical analysis, trade tactics, principles of decision-making process]. Izd-vo Politekhn. un-ta [Publishing house of the Polytechnical University], St. Petersburg

Lefevre E (1923) Reminiscences of a Stock Operator. George H. Doran and Company, New York.

Leland H (1980) Who should buy portfolio insurance? J. Finance Vol. 35 №2:581–594

Mandelbrot B (1960) The Pareto-Lévy Law and the Distribution of Income. Int. Econ. Rev. 1(2):79–106

Mandelbrot B (1961) Stable Paretian Random Functions and the Multiplicative Variation of Income. Econometrica 29(4):517–543

Mehra R, Prescott R (1985) The Equity Premium: A Puzzle. J. Monetary Econ. Vol. 15 №2:145–161

Merton R (1974) On the Pricing of Corporate Debt: the Risk Structure of Interest Rates. J. Finance 29(2):449–470

Peters E (1989) Fractal Structure in the Capital Markets. Financ. Anal. J. 45(4):32–37

Schumpeter J. (1983) [1934] The theory of economic development: an inquiry into profits, capital, credit, interest, and the business cycle. New Brunswick, New Jersey. Translated from the 1911 original German, Theorie der wirtschaftlichen Entwicklung.

Soros G (1987) The Alchemy of Finance. Reading the Mind of the Market. Wiley, Hoboken, New Jersey

Vaga T (1990) The Coherent Market Hypothesis. Financ. Anal. J. 46(6):36–49

Yuan K (2005) Asymmetric Price Movements and Borrowing Constraints: A Rational Expectations Equilibrium Model of Crises, Contagion, and Confusion. J. Finance 1:379–411